Simplex Method

We can solve any maximisation problem by using simplex method. Let us take an example.

Max : $Z = 8x + 6Y$.

s.t $4x + 2Y \leq 60$

$2x + 4Y \leq 48$

$x, Y \geq 0$.

First, we write this LPP in equality form by adding surplus variables.

$$Z - 8x - 6Y + 0 \cdot S_1 + 0 \cdot S_2 = 0$$
$$4x + 2Y + 1 \cdot S_1 + 0 \cdot S_2 = 60$$
$$2x + 4Y + 0 \cdot S_2 + 1 \cdot S_2 = 48$$

In table we can write the coefficients as →.

Table : 1

Rows.	Z	x_1	Y	S_1	S_2	Constant
1	1	-8	-6	0	0	0
2	0	4	2	1	0	60
3	0	2	4	0	1	48.

In 1st row, the column cotains highest value with the negative sign is known as pivot column.

Here. x column is pivot column.

Now, to select pivot row we have to take min $\left\{ \frac{60}{4}, \frac{48}{2} \right\}$ i.e row 2 is pivot row. The intersection of pivot row and pivot column i.e 4 is the pivot element.

Now in pivot row, we can do multiply and division. But in other two rows, we can only do addition and substraction.

First we have to 1 the pivot element and then other two values in pivot row to 0. i.e identity form. in the way that constant can never be negative.

This will end only when 1st row contain all the +ve values.

The solution gives the identity form and in the constant column.

Table 2

Rows	z	x	y	s_1	s_2	Constant
4	1	0	−2	2	0	120
5	0	1	1/2	1/4	0	15
6	0	0	(3)	−1/2	1	18

Row 5 = Row 2 ÷ 4

Row 4 = Row 1 + (8 × Row 5)

Row 6 = Row 3 − (2 × Row 5)

In similar manner y column is the pivot column and the pivot row is $\min\left\{ \frac{15}{1/2}, \frac{18}{3} \right\}$ i.e. row 6 & 3 is the pivot element

Table : 3

Rows	z	x	y	s_1	s_2	Constant
7	(1)	0	0	5/3	2/3	132
8	0	(1)	0	1/3	−1/6	12
9	0	0	(1)	−1/6	1/3	6

$2 - 1/3$

$\frac{1}{4} + \frac{1}{12}$

$= \frac{3+1}{12}$

Row 9 = Row 6 ÷ 3

Row 7 = Row 4 + 2 × Row 9

Row 8 = Row 5 − (Row 9/2)

Table 3 is the final table and gives the solution of LPP as Row 7 contains all +'ve values.

The solution of LPP is,

$Z^* = 132$

$\overline{x} = 12$

$\overline{y} = 6$

From this final table we can also find the solution of dual problem.

Dual Problem

Min : $Z' = 60x_1 + 48x_2$

s.t. $4x_1 + 2x_2 \geq 8$

$2x_1 + 4x_2 \geq 6$

$x_1, x_2 \geq 0$

The 1st row of final table i.e Row 7 gives the solution of dual problem. i.e

$Z = Z' = 132$

$s_1 = \overline{x_1} = 5/3$

$s_2 = \overline{x_2} = 2/3$